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Review

Damping and vibration analysis of polar orthotropic annular plates with ER treatment

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Abstract

In this study, the finite element analysis of polar orthotropic annular plate with an electrorheological (ER) fluid core and constraining layer is investigated. The equations of motion of the polar orthotropic annular sandwich plate are derived by using the discrete layer finite element method. The extensional and shear moduli of the electrorheological fluid layer are described by the complex quantities. When applying an electric field, the rheological property of the electrorheological fluid materials, such as viscosity, plasticity, and elasticity can be changed. The damping of the sandwich system is more effective when the electric fields are applied on the system. The ER fluid core is found to have a significant effect on the vibrational behaviors of the sandwich annular plate and the characteristics of the sandwich system can be controlled actively. Additionally, the effects of ER layer thickness, base annular plate stiffness, and some designed parameters on natural frequencies and modal loss factors are also discussed in this paper.

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1. Introduction

The sandwich structures with viscoelastic core layer have high damping capacity to suppress vibration and noise. The damping treatment can be applied easily to the mechanical structures and provide the major damping effects in vibrations. The pioneering work on the sandwich structure with viscoelastic core was presented by DiTaranto [1] and Mead and Markus [2]. They discussed the axial and bending vibration of the sandwich beam structures with arbitrary boundary conditions. Rao [3] studied the vibration analysis of the short sandwich beams. The vibration and damping characteristics of laminated composite beams by using Timoshenko beam element were studied by Rikards et al. [4].

The circular and annular plate had received a great deal of attention for the widely uses in many mechanical applications and the study on the circular and annular plate had been discussed by many researchers. Pandalai and Patel [5] studied the natural frequency of polar orthotropic circular plate for specific conditions. Then, Vijayakumar, Ramaiah [6] and Narita [7] investigated the natural frequencies of the polar orthotropic circular and annular plates by using the Rayleigh–Ritz method. Lin and Tseng [8] studied the free vibration problems of polar orthotropic circular and annular plates. Recently, many investigations of the vibration and damping analysis for the mechanical structures can be found. Mirza and Singh [9] investigated the axisymmetric vibration of the sandwich circular plate. Roy and Ganesan [10] presented the finite element method to calculate the vibration and damping analysis of circular plate with constrained layer treatment. Yu and Huang [11] studied the problem of the three-layered circular and annular plate based on the thin shell theory to discuss the characteristics of the viscoelastic layer.

Recently, many studies on the active control of the structural vibration had been devoted to the use of the electrorheological (ER) material. The material properties of the ER material can vary with respect to applied electric field and the damping of the ER fluid had been paid much attention by many researchers since Brooks et al. [12] studied the viscoelastic properties of the ER fluid. Choi and Park [13] carried out the investigation on the active vibration control and damping analysis of the cantilever sandwich beam with ER fluid. Then, Yalcintas and Coulter [14] adopted the ER material as controllable damping layer for the beam and plate configuration incorporating embedded sensors and control mechanism. The dynamic characteristics and damping effects of the sandwich isotropic and orthotropic rectangular plate structures were presented by Yeh and Chen [15,16].

In this paper, the vibration and damping behaviors of the sandwich polar orthotropic annular plate with ER core layer are studied. There are no works have been done to investigate this sandwich system with ER damping treatment to author's knowledge. The vibration and damping characteristics of the polar orthotropic annular plate with ER core layer are calculated by using the discrete layer finite element method. The extensional and shear moduli of the ER material are described by complex quantities and the natural frequencies and the modal loss factors of the sandwich system are obtained by solving the complex eigenvalue problem. Additionally, the effects of the ER layer and the influences of various parameters, such as thickness and applied electric fields, are also discussed in this study.

2. Problem formulation

2.1. Equation of motion

In Fig. 1, the sandwich polar orthotropic annular plate with ER core layer is considered. Layer 1 is the constraining layer and assumed to be pure elastic and polar orthotropic. The ER core layer is designed as layer 2 and the material properties can be changed by applying various electric fields. Layer 3 is the base annular plate and assumed to be undamped, pure elastic, and polar orthotropic. The base annular plate is designed as the inner radius $r_i = a$ and outer radius $r_0 = b$. Besides, the thickness for each layer is h_1, h_2, and h_3, respectively. In addition, the following assumptions must be mentioned first. The transverse displacements of each layer are equal and there are no slipping between the constraining-ER layer and ER-plate layer (Fig. 2).

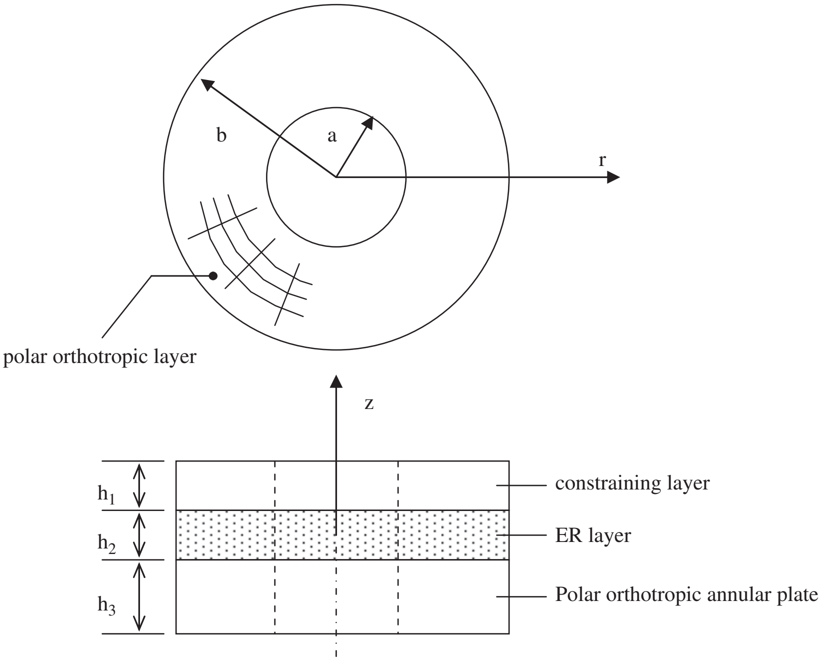

Fig. 1. Polar orthotropic annular plate with ER layer and constraining layer treatment.

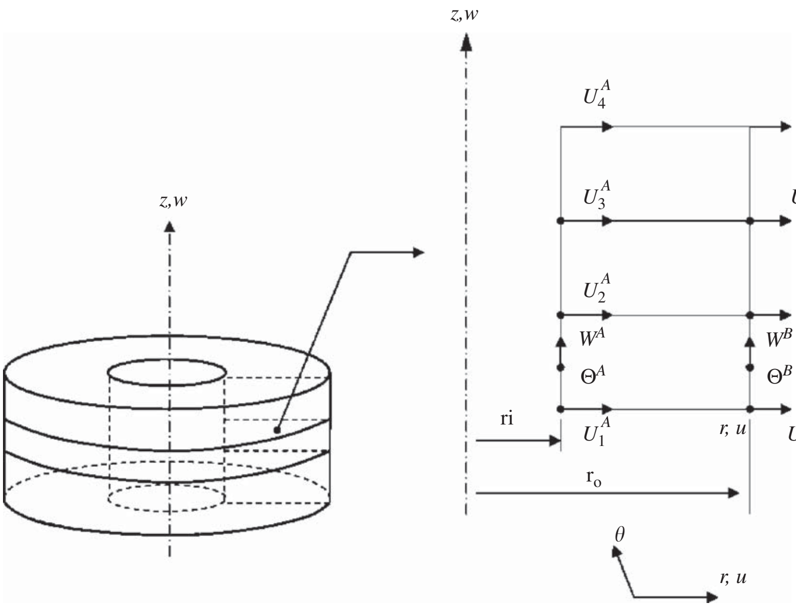

Fig. 2. Discrete layer annular finite element for three-layer element.

In order to analyze the problem, the displacement field of the layer i is employed as follows:

$$d_i = \begin{Bmatrix} u_i(r,\theta,z,t) \\ w_i(r,\theta,z,t) \end{Bmatrix} = \begin{bmatrix} \left(\frac{1}{2}-\frac{z}{h_i}\right) & \left(\frac{1}{2}+\frac{z}{h_i}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} U_i(r,\theta,t) \\ U_{i+1}(r,\theta,t) \\ W(r,\theta,t) \end{Bmatrix} = H_{1,i}(z) \begin{Bmatrix} U_i(r,\theta,t) \\ U_{i+1}(r,\theta,t) \\ W(r,\theta,t) \end{Bmatrix} \tag{1}$$

By using the interpolation in r-direction and the circumferential wave number m, the displacements of the interfaces for two-layers can be shown in terms of the nodal degrees of freedom as follows:

$$\begin{Bmatrix} U_i(r,\theta,t) \\ U_{i+1}(r,\theta,t) \\ W(r,\theta,t) \end{Bmatrix} = \begin{bmatrix} \phi_u^A & 0 & 0 & 0 & \phi_u^B & 0 & 0 & 0 \\ 0 & \phi_u^A & 0 & 0 & 0 & \phi_u^B & 0 & 0 \\ 0 & 0 & \phi_w^A & \phi_\Theta^A & 0 & 0 & \phi_w^B & \phi_\Theta^B \end{bmatrix} q_i^e(t) = H_2(r) q_i^e(t) \tag{2}$$

where the vector of the nodal displacements of the element $q_i^e(t) = \{U_i^A \;\; U_{i+1}^A \;\; W^A \;\; \Theta^A \;\; U_i^B \;\; U_{i+1}^B \;\; W^B \;\; \Theta^B\}^{\mathrm{T}}$, and $H_2(r)$ is the interpolation matrix and in which $\phi_u^A = (1-\xi)\cos m\theta$, $\phi_u^B = \xi \cos m\theta$, $\phi_w^A = (1-3\xi^2+2\xi^3)\cos m\theta$, $\phi_w^B = (3\xi^2-2\xi^3)\cos m\theta$, $\phi_\Theta^A = (\xi - 2\xi^2 + \xi^3)\cos m\theta$, $\phi_\Theta^B = (-\xi^2+\xi^3)\cos m\theta$, $\xi = (r-r_i)/(r_0-r_i)$.

Then, the strain–displacement relation for the ith layer of the system can be expressed as the following form:

$$\varepsilon_i = \begin{bmatrix} \dfrac{\partial}{\partial r} & 0 \\ \dfrac{1}{r} & 0 \\ \dfrac{\partial}{\partial z} & \dfrac{\partial}{\partial r} \end{bmatrix} d_i = D d_i \tag{3}$$

where D is the differential operator matrix, and $\varepsilon_i = \{\varepsilon_{r,i} \;\; \varepsilon_{\theta,i} \;\; \gamma_{rz,i}\}^{\mathrm{T}}$.

Then, the stress–strain relation can be obtained and shown as follows:

$$\sigma_i = C_i \varepsilon_i \tag{4}$$

where $\sigma_i = \{\sigma_{r,i} \;\; \sigma_{\theta,i} \;\; \tau_{r\theta,i}\}^{\mathrm{T}}$, C_i is the elasticity matrix and listed in Appendix B.

2.2. Energy and finite element formulation

According to the above equation, the kinetic and strain energies of the element for ith layer can be expressed as the following form:

$$T_i^e = \frac{1}{2}\int_V \rho_i \dot{d}_i^{\mathrm{T}} \dot{d}_i \, \mathrm{d}V \tag{5a}$$

$$V_i^e = \frac{1}{2}\int_V \rho_i \sigma_i^{\mathrm{T}} \varepsilon_i \, \mathrm{d}V \tag{5b}$$

where ρ_i is the mass density of the ith layer.

The kinetic and strain energies of the element can be rewritten as follows by substituting Eqs. (1)–(4):

$$V_i^e = \tfrac{1}{2} U_i^{e\mathrm{T}} K_i^e U_i^e \tag{6a}$$

$$T_i^e = \tfrac{1}{2} \dot{U}_i^{e\mathrm{T}} M_i^e \dot{U}_i^e \tag{6b}$$

in which

$$K_i^e = \int_V \rho_i (H_{1,i} H_2)^{\mathrm{T}} (H_{1,i} H_2) \, \mathrm{d}V \tag{6c}$$

$$M_i^e = \int_V (D H_{1,i} H_2)^{\mathrm{T}} C_i^{\mathrm{T}} (D H_{1,i} H_2) \, \mathrm{d}V \tag{6d}$$

Then, the following relations must be obtained by combining the elemental matrices into the global stiffness and mass matrices:

$$U_i^e = Tr_i^e U \tag{7}$$

where U and Tr_i^e are the global nodal coordinate vector and transformation matrix, respectively.

The equation of motion for the polar orthotropic annular sandwich system can be expressed as the following form by assembling the contributions of all elements:

$$M\ddot{U} + KU = 0 \tag{8a}$$

in which

$$K = \sum_{i=1}^{3}\left(\sum_{e=1}^{N_i} Tr_i^{e\mathrm{T}} K_i^e Tr_i^e\right) \tag{8b}$$

$$M = \sum_{i=1}^{3}\left(\sum_{e=1}^{N_i} Tr_i^{e\mathrm{T}} M_i^e Tr_i^e\right) \tag{8c}$$

where N_i is the element number of the ith layer.

Finally, the complex eigenvalues $\tilde{\lambda}$ of the above complex eigenvalue problems can be calculated numerically. The natural frequencies ω and modal loss factor η_v of the sandwich polar orthotropic annular plate with ER core layer can be obtained as follows:

$$\omega = \sqrt{\mathrm{Re}(\tilde{\lambda})} \tag{9a}$$

$$\eta_v = \frac{\mathrm{Im}(\tilde{\lambda})}{\mathrm{Re}(\tilde{\lambda})} \tag{9b}$$

3. Numerical results

In this paper, the discrete layer finite element method is adopted to calculate the vibration problem of the sandwich system with ER core treatment. In order to validate the results and analysis, the solutions of the natural frequency and modal loss factor for present model and references are presented. In Table 1, the natural frequencies for the polar orthotropic annular plate are obtained and the boundary conditions are free at inner radius and clamped at outer radius. Besides, the calculations of the natural frequencies and modal loss factors for the sandwich annular plate are also presented in Table 2, and the boundary conditions are clamped at inner radius and free at outer radius. Good accuracy and convergence can be found in the above comparisons.

The damping effects of the sandwich system are provided by the ER fluid in this study, and only the electric field dependence of ER fluid need to consider based on the existing model of ER material. Therefore, the complex modulus of ER fluid can be simplified into the following form, which was experimentally calculated by Don [17]:

$$G_2(E_*) = G' + G'' \tag{10a}$$

Table 1
Comparisons between published and proposed methods for polar orthotropic annular plate.

b/h	Non-dimensional fundamental natural frequency			
	$b/a = 0.1$		$b/a = 0.5$	
	Present	Ref. [8]	Present	Ref. [8]
10	13.471	13.526	20.818	20.636
20	14.042	13.936	21.959	21.851
50	14.238	14.147	22.318	22.233
100	14.267	14.178	22.371	22.290

Table 2
Comparisons between published and proposed methods for the full coverage annular plate.

Mode (n, m)	Natural frequency (Hz)		Modal loss factor	
	Present	Ref. [10]	Present	Ref. [10]
(0, 0)	74.44	74.38	0.1128	0.1127
(0, 1)	73.00	73.08	0.09542	0.09576
(0, 2)	96.20	96.38	0.1016	0.1021
(0, 3)	144.0	142.8	0.1210	0.1212
(0, 4)	205.2	203.7	0.1170	0.1177

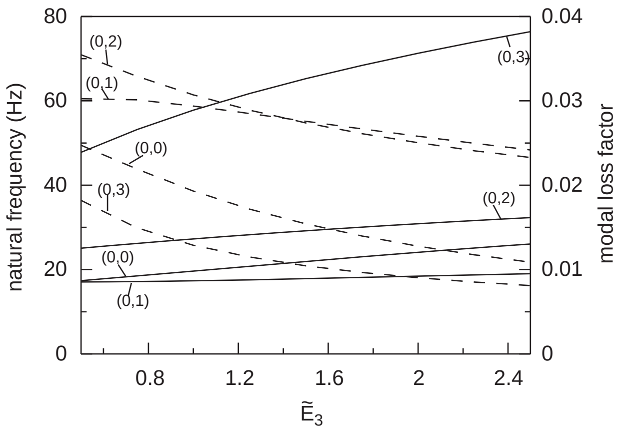

Fig. 3. Effects of $\tilde{E}_3$ on the natural frequencies and the modal loss factors of the sandwich annular plate: key: ———, natural frequency; – – –, modal loss factor ($\tilde{E}_1 = 1$, $\tilde{a} = 0.1$, $E_* = 0.5\,\text{kV/mm}$, $b = 0.15\,\text{m}$, $h_3 = 0.5\,\text{mm}$, $\tilde{h}_{13} = 0.1$, $\tilde{h}_{23} = 0.5$).

in which

$$G' \approx 15\,000 E_*^2 \tag{10b}$$

$$G'' \approx 6900 \tag{10c}$$

where G' is the shear storage modulus, G'' is the loss modulus, E_* is the applied electric field in kV/mm (0–0.25 kV/mm).

Additionally, in order to simplify the following analysis and discussion, the geometric and non-dimensional parameters are used:

$$\tilde{a} = \frac{a}{b}, \quad \tilde{h}_{23} = h_2/h_3, \quad \tilde{h}_{13} = h_1/h_3, \quad \tilde{E}_1 = \frac{E_{\theta,1}}{E_{r,1}}, \quad \tilde{E}_3 = \frac{E_{\theta,3}}{E_{r,3}}, \quad \upsilon_2 = 0.49,$$

$$\upsilon_{\theta r,1} = \upsilon_{\theta r,3} = 0.29, \quad \rho_1 = \rho_3 = 2700\,\text{kg/m}^3, \quad \rho_2 = 1700\,\text{kg/m}^3, \quad b = 0.15\,\text{m},$$

$E_{r,1} = E_{r,3} = 70\,\text{GPa}$, $\kappa = \pi^2/12$ (for layer 1, 3), $\kappa = 1$ (for layer 2).

Fig. 3 shows the effects of $\tilde{E}_3$ on the natural frequency and modal loss factor of the polar orthotropic annular plate with ER core layer for modes (0,0), (0,1), (0,2), and (0,3). The boundary conditions of the sandwich system are clamped at inner radius and free at outer radius. Then, the effects of applied electric fields on the natural frequency and modal loss factor of the polar orthotropic sandwich annular plate are shown in Fig. 4.

The effects of $\tilde{E}_3$ on the natural frequency and modal loss factor of the sandwich system with various thicknesses of ER layer are plotted in Fig. 5. The numerical results for modes (0,0) and (0,1) are shown in Fig. 5(a) and (b), respectively. For modes (0,0) and (0,1), the analytical results are similar according to the above results.

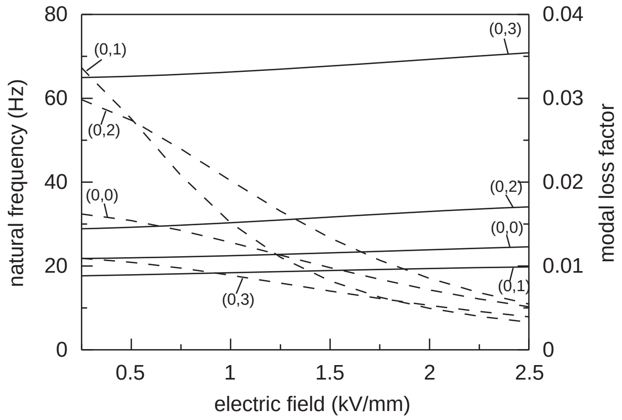

Fig. 4. Effects of electric fields on the natural frequencies and the modal loss factors of the sandwich annular plate: key: ———, natural frequency; – – –, modal loss factor ($\tilde{E}_1 = 1$, $\tilde{E}_3 = 1.5$, $\tilde{a} = 0.1$, $b = 0.15\,\mathrm{m}$, $h_3 = 0.5\,\mathrm{mm}$, $\tilde{h}_{13} = 0.1$, $\tilde{h}_{23} = 0.5$).

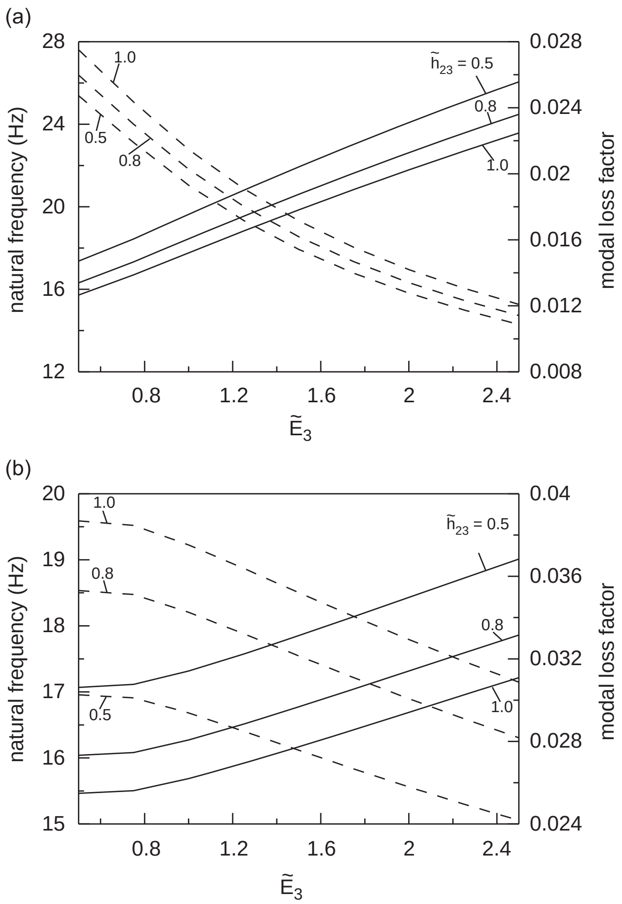

Fig. 5. Effects of $\tilde{E}_3$ on the natural frequencies and the modal loss factors of the sandwich annular plate with various thickness of ER layer: key: ———, natural frequency; – – –, modal loss factor ($\tilde{E}_1 = 1$, $\tilde{a} = 0.1$, $E_* = 0.5\,\mathrm{kV/mm}$, $b = 0.15\,\mathrm{m}$, $h_3 = 0.5\,\mathrm{mm}$, $\tilde{h}_{13} = 0.1$). (a) Mode (0,0) and (b) mode (0,1).

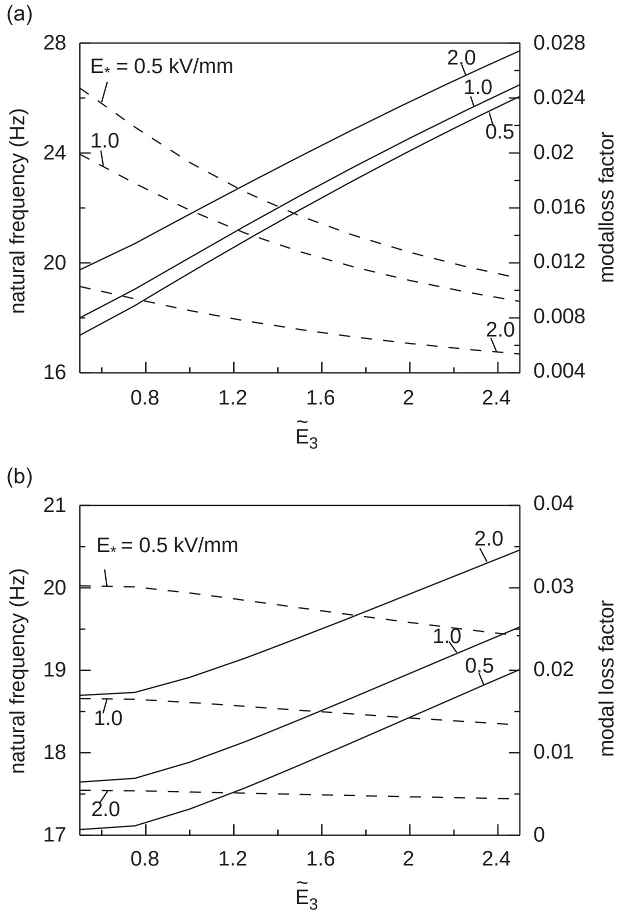

Fig. 6. Effects of $\tilde{E}_3$ on the natural frequencies and the modal loss factors of the sandwich annular plate with various applied electric fields: key: ———, natural frequency; – – –, modal loss factor ($\tilde{E}_1 = 1$, $\tilde{a} = 0.1$, $b = 0.15\,\text{m}$, $h_3 = 0.5\,\text{mm}$, $\tilde{h}_{13} = 0.1$, $\tilde{h}_{23} = 0.5$). (a) Mode (0,0) and (b) mode (0,1).

Fig. 6 shows the numerical results of the effects of $\tilde{E}_3$ on the natural frequency and modal loss factor of the sandwich system with various electric fields.

The effects of thickness of ER layer on the natural frequency and modal loss factor with various ratios $\tilde{E}_3$ are presented in Fig. 7. Fig. 8 shows the variations of the natural frequency and modal loss factor of the sandwich system.

The effects of electric field on the natural frequency and modal loss factor of the sandwich system with various ratios $\tilde{a}$ are plotted in Fig. 9. And, the effects of thickness of ER layer on the natural frequency and modal loss factor with various radius ratios are presented in Fig. 10.

4. Discussions

According to the numerical results in Fig. 3, the natural frequency will increase and the modal loss will decrease while $\tilde{E}_3$ increases. The larger $\tilde{E}_3$ will increase the stiffness of the system and decrease the damping effects of the system. Besides, the variations are similar for different modes in the figure. As shown in Fig. 4, it can be observed that the natural frequency increases as the applied electric field magnitude increases. Additionally, it also can be found that the modal loss factor of the system decreases as the applied electric field increases. The tendency for each mode is the same from above analytical results. The variations are because the larger applied electric fields will increase the stiffness of the ER layer and decrease the damping effects of

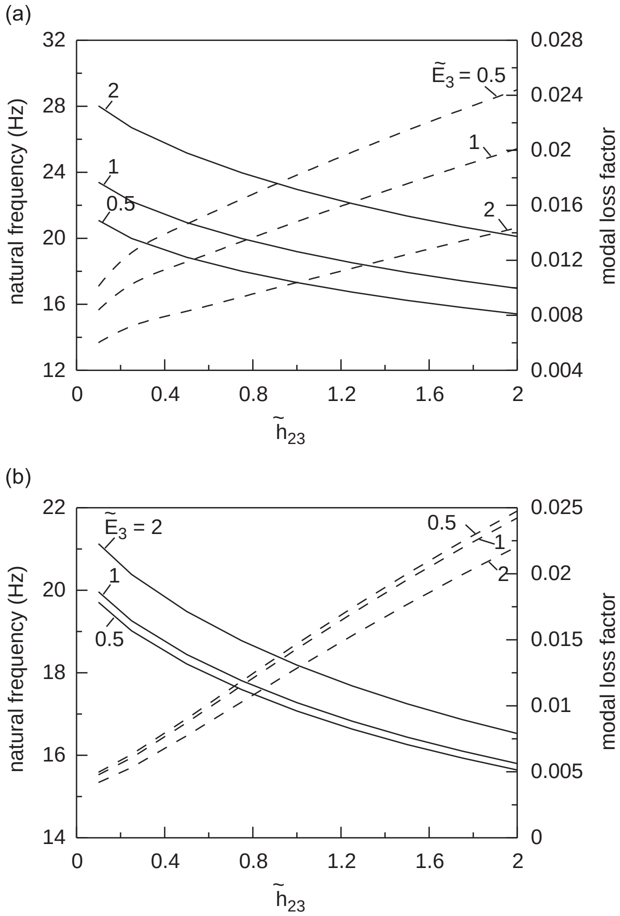

Fig. 7. Effects of thickness of ER layer on the natural frequencies and the modal loss factors of the sandwich annular plate with various ratios $\tilde{E}_3$: key: ———, natural frequency; – – –, modal loss factor ($\tilde{E}_1 = 1$, $\tilde{a} = 0.1$, $E_* = 1.5\,\mathrm{kV/mm}$, $b = 0.15\,\mathrm{m}$, $h_3 = 0.5\,\mathrm{mm}$, $\tilde{h}_{13} = 0.1$). (a) Mode (0,0) and (b) mode (0,1).

the ER layer for the sandwich system. From the above results of Figs. 3 and 4, the vibration characteristics of the sandwich system can be changed by the applied electric fields and strength parameter $\tilde{E}_3$. Therefore, those parameters can be used to control and change the natural frequency and modal loss factor of the sandwich system.

In Fig. 5, it can be observed that the natural frequency becomes larger when $\tilde{E}_3$ increases. In addition, the modal loss factor will decrease as $\tilde{E}_3$ increases. On the other hand, the natural frequency decreases and modal loss factor increases when the thickness of ER layer increases. The thickness of the ER layer will affect the stiffness and damping effects of the sandwich system and those characteristics can be controlled by the applied electric fields. In practical applications, we can change natural frequency and the modal loss factor of the system by the thickness of ER fluid core and $\tilde{E}_3$. From Fig. 6, it can be observed that the natural frequency increases as the applied electric fields increase. On the contrary, the modal loss factor decreases while the applied electric field increases because of the decrement of the damping effects of the ER layer of the sandwich system. According to the results, the applied electric field can change and control those dynamic characteristics of the sandwich system immediately.

From the results in Fig. 7, it can be found that the larger thickness of the ER layer, the smaller natural frequency. Contrary to the modal loss factor, the larger thickness of ER layer, the larger modal loss factor. It is because that the larger thickness of the ER layer will increase the damping effects of the system and decrease the stiffness of the sandwich system. Besides, the natural frequency will increase and modal loss factor will

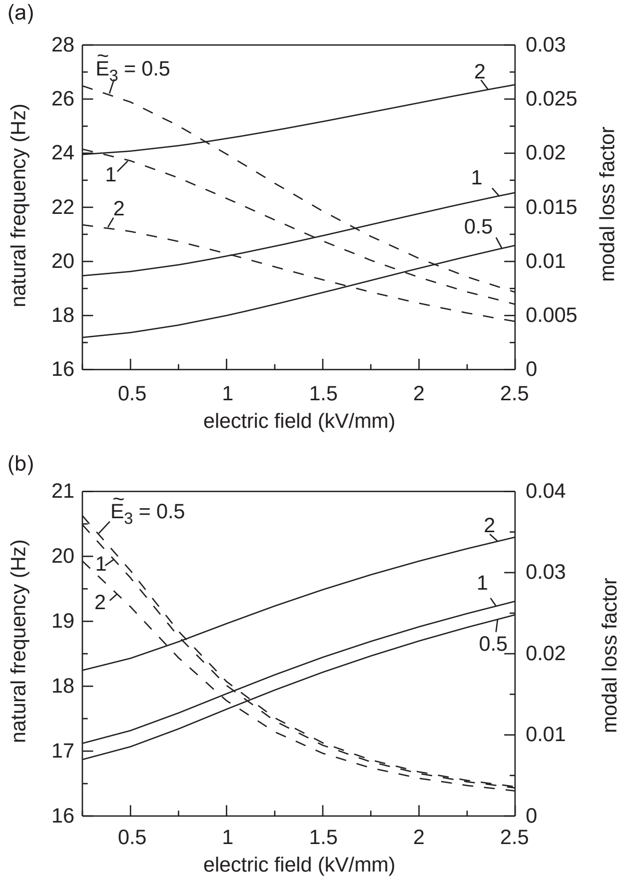

Fig. 8. Effects of electric fields on the natural frequencies and the modal loss factors of the sandwich annular plate with various ratios $\tilde{E}_3$: key: ———, natural frequency ; – – –, modal loss factor ($\tilde{E}_1 = 1$, $\tilde{a} = 0.1$, $b = 0.15\,\text{m}$, $h_3 = 0.5\,\text{mm}$, $\tilde{h}_{13} = 0.1$). (a) mode (0,0) and (b) mode (0,1).

decrease while $\tilde{E}_3$ increases. The thickness of ER fluid core layer has significant influence on the natural frequency and modal loss factor and we can design some controllable device by various thicknesses of ER fluid core layer. To base on the results in Fig. 8, the natural frequency increases while the applied electric field increases. As to the modal loss factor, it can be seen that the modal loss factor will decrease when the applied electric field increases. The natural frequency increases and the modal loss factor decreases because of the increment of the stiffness and decrement of the damping effects of the ER layer by the applied electric fields. From the results, the natural frequency and modal loss factor can be controlled and changed rapidly and easily by applying various electric fields.

In Fig. 9, it can be observed that the larger applied electric fields, the larger natural frequency and smaller modal loss factor. And, as the radius ratio $\tilde{a}$ increases, the natural frequency will increase according to the results. Furthermore, modal loss factor of the system will decrease as the radius ratio increases. In Fig. 10, the results show that the larger thickness of ER layer, the smaller natural frequency and larger modal loss factor. From the numerical results, it can be seen that the larger radius ratio, the larger natural frequency. On the contrary, the modal loss factor will decrease when the radius ratio increases. For modes (0,0) and (0,1), the analytical tendency is similar from the above results.

From the above numerical results, we can find that the thickness of ER fluid core layer, the applied electric fields, and the parameters $\tilde{a}$ and $\tilde{E}_3$ can be used to change and control the natural frequency and modal loss factor of the sandwich system. In practical applications, increasing the natural frequency can avoid the resonance of the system and increase the stability of the sandwich system. Besides, increasing the modal

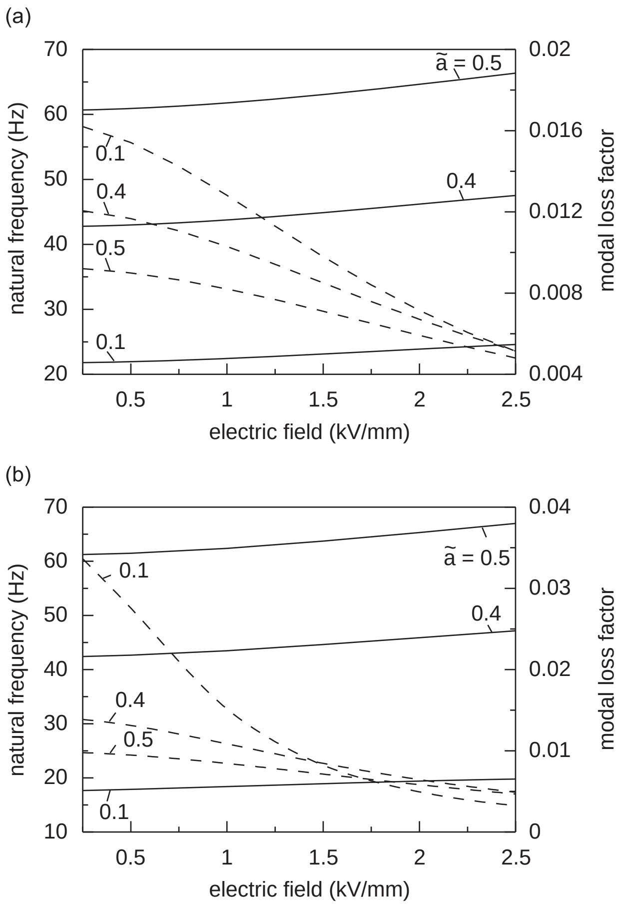

Fig. 9. Effects of electric fields on the natural frequencies and the modal loss factors of the sandwich annular plate with various ratios $\tilde{a}$: key: ———, natural frequency; – – –, modal loss factor ($\tilde{E}_1 = 1$, $\tilde{E}_3 = 1.5$, $b = 0.15\,\mathrm{m}$, $h_3 = 0.5\,\mathrm{mm}$, $\tilde{h}_{13} = 0.1$, $\tilde{h}_{23} = 0.5$). (a) mode (0,0) and (b) mode (0,1).

loss factor can absorb the vibration of the system to decrease the generation of the system vibration. Those designed parameters can be used and designed according to the different requirements of the devices. Furthermore, the vibration characteristics of the designed devices can be controlled and changed actively and immediately.

5. Conclusions

In this study, the vibration and damping problems of the polar orthotropic sandwich annular plate with ER core layer are investigated and the discrete layer finite element method are adopted to calculate the problems. Besides, the complex description of viscoelastic material is used for the ER fluid. It can be observed that the ER damping treatment can make the system stable from the analytical results. And, the results show that the natural frequency and modal loss factor will vary with changing the strength ratio $\tilde{E}_3$.

On the other hand, the applied electric field can change the vibration and damping characteristics of the polar orthotropic sandwich system. Then, the thickness of the ER layer also can change the stiffness of the sandwich system, and the natural frequency and modal loss factor of the system will change. Besides, the damping effects of the ER layer can be changed by applying different electric fields and shown to have significant variations on the vibration and damping characteristics.

The present analytical results can be used to design the smart devices and other mechanical applications as basic information. The mechanical devices, such as CD-ROM devices, can be controlled and changed actively

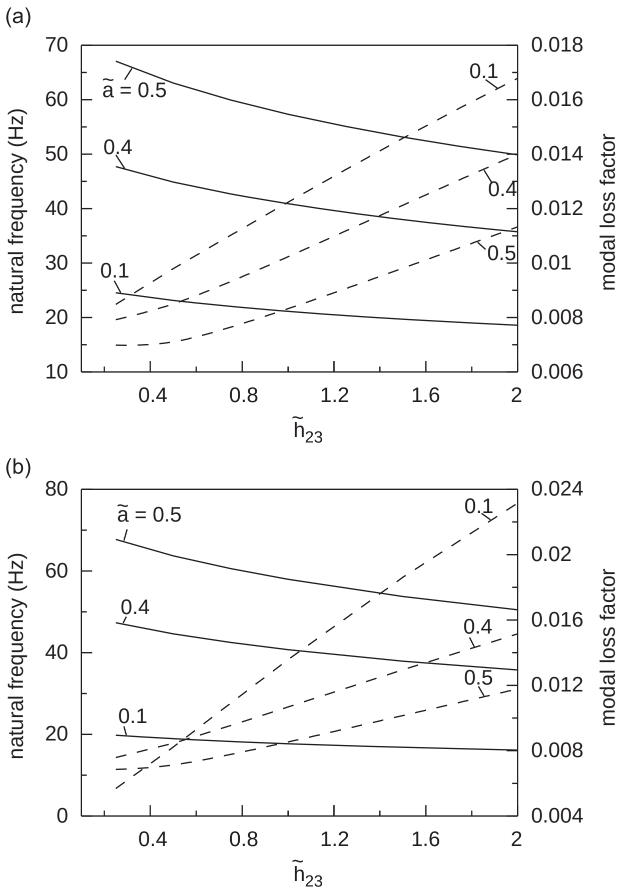

Fig. 10. Effects of thickness of ER layer on the natural frequencies and the modal loss factors of the sandwich annular plate with various ratios $\tilde{a}$: key: ———, natural frequency; – – –, modal loss factor ($\tilde{E}_1 = 1$, $\tilde{E}_3 = 1.5$, $E_* = 1.5\,\text{kV/mm}$, $b = 0.15\,\text{m}$, $h_3 = 0.5\,\text{mm}$, $\tilde{h}_{13} = 0.1$). (a) mode (0,0) and (b) mode (0,1).

by those parameters. And, the dynamic stability problems of the polar orthotropic system with ER material are the interesting topics to be investigated in the future.

Appendix A

Notation

a	inner radius
b	outer radius
d_i	displacement field of the layer i
D	the differential operator matrix
E_*	the applied electric field
$E_{r,i}$, $E_{\theta,i}$	Young's modulus of layer i, $i = 1, 3$
G'	the shear storage modulus of ER fluid
G''	the loss modulus of ER fluid
$G_2(E_*)$	the complex modulus of ER fluid
h_i	thickness of layer i, $i = 1, 2, 3$
$H_{1,i}(z)$	the transverse interpolation matrix of layer i

$H_2(r)$	the interpolation matrix
N_i	the element number of the *i*th layer
η_v	the modal loss factor
κ	the shear correction factor
$\tilde{\lambda}$	complex eigenvalue
ρ_i	the mass density of the *i*th layer
$\upsilon_{r\theta,i}$, $\upsilon_{\theta r,i}$	Poisson's ratio of layer i, $i = 1, 3$
ω	the natural frequency

Appendix B

$$C_i = \begin{bmatrix} C_{11,i} & C_{12,i} & 0 \\ C_{21,i} & C_{22,i} & 0 \\ 0 & 0 & C_{44,i} \end{bmatrix}$$

for isotropic ER material, $C_{11,2} = C_{22,2} = E_2/1 - \upsilon_2^2$, $C_{12,2} = C_{21,2} = \upsilon_2 E_2/1 - \upsilon_2^2$, $C_{44,2} = E_2/2(1 + \upsilon_2)$, $\upsilon_2 = 0.499$, respectively.

For polar orthotropic material ($i = 1,3$), $C_{11,i} = E_{r,i}/1 - \upsilon_{r\theta,i}\upsilon_{\theta r,i}$, $C_{22,i} = E_{\theta,i}/1 - \upsilon_{r\theta,i}\upsilon_{\theta r,i}$, $C_{12,i} = C_{21,i} = \upsilon_{\theta r,i}E_{r,i}/1 - \upsilon_{r\theta,i}\upsilon_{\theta r,i}$, $C_{44,i} = \kappa G_{rz,i}$, respectively. In the above equations, E_i is Young's modulus, υ_i is the Poisson's ratio, and κ is the shear correction factor.

References

[1] R.A. DiTaranto, Theory of vibratory bending for elastic and viscoelastic layered finite-length beams, *ASME Journal of Applied Mechanics* (1965) 881–886.

[2] D.J. Mead, S. Markus, The forced vibration of a three-layer, damped sandwich beam with arbitrary boundary conditions, *AIAA Journal* 10 (2) (1969) 163–175.

[3] D.K. Rao, Vibration of short sandwich beams, *Journal of Sound and Vibration* 52 (1977) 253–263.

[4] R. Rikards, A. Chate, E. Barkanov, Finite element analysis of damping the vibrations of laminated composites, *Composite Structures* 47 (1993) 1005–1015.

[5] K.A.V. Pandalai, S.A. Patel, Natural frequencies of orthotropic circular plates, *AIAA Journal* 3 (1965) 780–781.

[6] K. Vijayakumar, G.K. Ramaiah, Estimation of higher natural frequencies of polar orthotropic annular plates, *Journal of Sound and Vibration* 32 (1974) 265–278.

[7] Y. Narita, Vibration of continuous polar orthotropic annular and circular plates, *Journal of Sound and Vibration* 93 (1984) 503–511.

[8] C.C. Lin, C.S. Tseng, Free vibration of polar orthotropic laminated circular and annular plates, *Journal of Sound and Vibration* 209 (1998) 797–810.

[9] S. Mirza, A.V. Singh, Axisymmetric vibration of circular sandwich plates, *AIAA Journal* 12 (1974) 1418–1420.

[10] P.K. Roy, N. Ganesan, A vibration and damping analysis of circular plates with constrained damping layer treatment, *Computers Structures* 49 (1993) 269–274.

[11] S.C. Yu, S.C. Huang, Vibration of a three-layered viscoelastic sandwich circular plate, *International Journal of Mechanical Science* 43 (2001) 2215–2236.

[12] D.A. Brooks, J. Goodwin, C. Hjelm, L. Marshall, C. Zukoski, Viscoelastic studies on an electro-rheological fluid, *Colloids and Surfaces* 18 (1986) 293–312.

[13] S.B. Choi, Y.K. Park, Active vibration control of cantilever beam containing an electro-rheological fluid, *Journal of Sound and Vibration* 172 (1994) 428–432.

[14] M. Yalcintas, J.P. Coulter, Analytical modeling of electrorheological material based adaptive beams, *Journal of Intelligent Material Systems and Structures* 6 (1995) 488–497.

[15] J.Y. Yeh, L.W. Chen, Dynamic stability of a sandwich plate with a constraining layer and electrorheological fluid core, *Journal of Sound and Vibration* 285 (2005) 637–652.

[16] J.Y. Yeh, L.W. Chen, Finite element dynamic analysis of orthotropic sandwich plates with an electrorheological fluid core layer, *Composite Structures* 78 (3) (2007) 368–376.

[17] D.L. Don, An investigation of electrorheological material adoptive structures, Master's Thesis, Lehigh University, Bethlehem, Pennsylvania, 1993.